A New Lumped-Parameter Approach to Simulating Flow Processes in Unsaturated Dual-Porosity Media

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Abstract: We have developed a new lumped-parameter dual-porosity approach to simulating unsaturated flow processes in fractured rocks. Fluid flow between the fracture network and the matrix blocks is described by a nonlinear equation that relates the imbibition rate to the local difference in liquid-phase pressure between the fractures and the matrix blocks. This equation is a generalization of the Warren-Root equation, but, unlike the Warren-Root equation, is accurate in both the early and late time regimes. The fracture/matrix interflow equation has been incorporated into a computational module, compatible with the TOUGH simulator, to serve as a source/sink term for fracture elements. The new approach achieves accuracy comparable to simulations in which the matrix blocks are discretized, but typically requires an order of magnitude less computational time.

Introduction

In a dual-porosity medium such as a fractured/porous rock mass, an interconnected network of fractures provides most of the global permeability, whereas most of the fluid storage takes place in the relatively low-permeability matrix blocks. In principle, one way to model flow in a fractured/porous rock mass would be to explicitly account for each fracture and each matrix block in the computational mesh. In practice this is rarely possible, due to the large number of gridblocks that would be needed, and to the difficulty in gathering the necessary information concerning the locations, lengths, and individual transmissivities of the fractures.

Another approach, which is more tractable, is the MINC method (Pruess and Narasimhan, 1985). In this approach, the fracture network is treated as a continuum, which is then discretized into fracture gridblocks. The matrix blocks that are located in the region occupied by each fracture gridblock are then represented by a single nested set of shell-like elements. This usually requires many fewer elements than would be needed in a discrete-fracture simulation, but still requires about 5-10 times as many matrix elements as fracture elements.

One way to eliminate the need for discretizing the matrix blocks is to treat them in a lumped-parameter manner, using only mean values of the pressure, saturation, etc. The matrix blocks then serve as source/sinks for the fracture elements. This approach was used by Barenblatt et al. (1960) and Warren and Root (1963), who assumed that the strength of the source/sink was proportional to the difference in potential between the fractures and matrix blocks. We have used a nonlinear modification of their approach, and implemented it into a computational module that is compatible with TOUGH. Our approach, which is summarized below, is also described in more detail by Zimmerman et al. (1995).

Warren-Root Method

In the traditional lumped-parameter approach (Barenblatt et al., 1960; Warren and Root, 1963) flow is assumed to take place through the fracture network, and also between the fractures and the matrix blocks. Each point in space has associated with it two sets of parameters, for the fractures and for the matrix blocks. The rate of flow between the fractures and matrix blocks must be expressed as some function of

the local fracture pressure, the local mean matrix pressure, and other properties such as the permeabilities, etc. In order to maintain the linearity of the equations, Barenblatt et al. (1960) and Warren and Root (1963) assumed that the volumetric flow rate of fluid from the fractures into the matrix blocks, per unit volume of matrix block, was governed by

$$q = \frac{\alpha k_m V_m}{\mu}(P_f - \bar{P}_m), \qquad (1)$$

where k_m is the permeability of the matrix block, μ is the viscosity of the fluid, P_f is the local pressure in the fractures, $\bar{P}_m$ is the mean pressure in the matrix block at a specified point in the fracture continuum, V_m is the volume of the matrix block, and α is a geometric factor that has dimensions of $[L^{-2}]$. The flowrate q given in eq. (1) therefore has dimensions of $[L^3 T^{-1}]$. The relationship between the numerical value of α and the size and shape of the matrix block is discussed by deSwaan (1990) and Zimmerman et al. (1993).

The flow of fluid into the matrix block causes the fluid pressure in the block to increase. If the fluid is a slightly-compressible liquid, this pressure increase is described by

$$\phi_m c_m V_m \frac{d\bar{P}_m}{dt} = q, \qquad (2)$$

where ϕ_m is the (dimensionless) porosity of the matrix block, and c_m, with dimensions of $[P^{-1}]$, is the combined compressibility of the pore fluid and the pore space of the matrix blocks. If eqs. (1) and (2) are combined, they yield the following equation that governs the fluid pressure in the matrix block:

$$\frac{d\bar{P}_m}{dt} = \frac{\alpha k_m}{\phi_m \mu c_m}(P_f - \bar{P}_m). \qquad (3)$$

For saturated flow of a slightly-compressible fluid, eq. (3) represents the most-slowly-decaying Fourier mode in the exact solution for infiltration into a matrix block (Zimmerman et al., 1993). As such, it correctly models the long-time, quasi-equilibrium regime of fracture/matrix interaction. However, as it does not contain any of the higher modes, it is not accurate at earlier stages of imbibition (deSwaan, 1990; Zimmerman et al., 1993). When incorporated into a dual-porosity model, the Warren-Root equation leads to qualitatively incorrect behavior during the transition between fracture-dominated and effective-continuum flow regimes (Najurieta, 1980).

Dykhuizen (1990) attempted to remedy this situation by essentially using two different equations for fracture/matrix flow, one in the early-time regime, and one in the late-time regime. Another related approach is that of Pruess and Wu (1993), who used the integral method to model flow in the matrix block using a polynomial pressure profile. This is not a lumped-parameter approach, but is similar in that only a small number of parameters are needed to account for the thermodynamic state of the matrix block, and the fracture/matrix flow interactions.

We use a variation of Dykhuizen's approach, wherein a single equation is found that seems to accurately represent the flowrate at both late and early times. This equation, which is derived from an approximate solution found by Vermeulen (1953) for diffusion into a spherical block, is

$$q = \frac{\alpha k_m V}{\mu} \frac{[(P_f - P_i)^2 - (\bar{P}_m - P_i)^2]}{2(\bar{P}_m - P_i)}, \qquad (4)$$

where P_i is the initial pressure in the matrix block, and α is the same shape factor that is used in the Warren-Root equation. When $\bar{P}_m$ is close to P_f, which is to say that the matrix and fractures are nearly in equilibrium with each other, eq. (4) reduces to eq. (1). However, eq. (4) is also accurate at early times, when the fracture pressure is varying rapidly and the matrix pressure has not yet had sufficient time to respond (see Zimmerman et al., 1993).

Unsaturated Flow Simulation Procedure

We have implemented a Vermeulen-type expression for fracture/matrix flow as part of a module of subroutines that is compatible with TOUGH (Pruess, 1987). The saturated flow version of this procedure is described by Zimmerman et al. (1993); here we focus on the unsaturated flow version.

TOUGH contains provisions for sources/sinks of mass and heat, which are calculated in the subroutine QU. The sources/sinks are typically used to account for fluid that is injected or withdrawn from a borehole that penetrates one of the gridblocks. We have modified this subroutine so as to include a new type of source/sink, which represents liquid water flowing into (or out of) the fracture gridblock from the matrix blocks

that are contained in a given fracture gridblock. The magnitude of the fracture/matrix flux for each fracture gridblock is computed using

$$q = \frac{\alpha V_m k_m k_r}{\mu} \frac{[(\psi_f - \psi_i)^2 - (\bar{\psi}_m - \psi_i)^2]}{2(\bar{\psi}_m - \psi_i)}, \quad (5)$$

where ψ is the capillary pressure, and k_r is the relative permeability of the matrix block. As the imbibition rate is primarily controlled by the hydraulic conductivity of the matrix block at the wetted boundary, we evaluate k_r at the capillary pressure that exists at the outer boundary of the matrix block, which is to say at the capillary pressure of the fracture. Note that the actual driving force for flow is the difference in liquid phase pressure; however, as the air phase pressure is assumed to be the same in the matrix and their adjacent fractures (i.e., the Richards approximation; see Zimmerman et al., 1995), the difference in liquid phase pressures is equivalent to the difference in capillary pressures. The volume V_m represents the total volume of matrix rock contained within the fracture gridblock; it is related to the volume of the fracture gridblock by $V_m = (1-\phi_f)V_f$, where $\phi_f \ll 1$ is the fracture porosity.

The generation term q represents the average instantaneous flux out of a given fracture gridblock, over the time interval $[t, t+\Delta t]$. To make the calculation fully implicit, the flux is computed using the values of the variables at time $t+\Delta t$. The new value of $\bar{\psi}_m$ that exists in the matrix block at time $t+\Delta t$ must be consistent with the new average saturation. From a mass balance in the matrix block, the new average saturation at time $t+\Delta t$ is given by

$$\bar{S}_m(t+\Delta t) = \bar{S}_m(t) + \frac{q\,\Delta t}{\phi_m V_m}. \quad (6)$$

The mean saturation $\bar{S}_m$ and the mean capillary pressure $\bar{\psi}_m$ are related to each other through the capillary pressure function of the matrix rock:

$$\bar{\psi}_m = f(\bar{S}_m), \quad (7)$$

where f is the capillary pressure function. At each TOUGH iteration, eqs. (5,6,7) are iterated (for each fracture gridblock) to find a consistent set of values of $\{\bar{\psi}_m, \bar{S}_m, q\}$. We must also calculate additional contributions to the Jacobian matrix, whose components are the partial derivatives of the energy, water and air residuals with respect to changes in the primary variables, which are the liquid saturation, the gas phase pressure, and the temperature. In our formulation, we neglect the possible presence of dissolved air in the water that flows between the fracture elements and their associated matrix blocks. However, we do include the latent heat that is transported with the liquid, which is calculated by multiplying the mass flux given by eq. (5) by the liquid phase enthalpy.

Example: Flow in a Leaky Fracture

As an example of the use of the new approach, consider the problem of water flowing along a single fracture imbedded in a porous matrix (Travis et al., 1984; Martinez, 1987; Nitao and Buscheck, 1991). Flow into the fracture is driven by the imposed potential at the inlet boundary, which we take to be zero in the following example. For the matrix blocks, we use the hydrological parameters that have been estimated for the Topopah Spring Member of the Paintbrush Tuff at Yucca Mountain (Rulon et al., 1986). In terms of the van Genuchten (1980) parameters, these properties are $k = 3.9\times10^{-18}\,\mathrm{m}^2$, $\phi = 0.14$, $n = 3.04$, $\alpha_{vg} = 1.147\times10^{-5}\,\mathrm{Pa}^{-1}$, $S_s = 0.984$, and $S_r = 0.318$. For the fracture, we use the properties that were derived by Pruess et al. (1988) using a mathematical model of a fracture as a rough-walled channel: $k = 5.5\times10^{-11}\,\mathrm{m}^3$ (per fracture), $n = 2.89$, $S_s = 1.0$, $S_r = 0.0$, and $\alpha_{vg} = 6.06\times10^{-4}\,\mathrm{Pa}^{-1}$. The volumes of the fracture elements were chosen to correspond to an aperture of $800\,\mu\mathrm{m}$. This relatively large aperture was chosen so as to accentuate the early-time regime of the solution, in which matrix imbibition is not yet of much consequence, so as to clearly verify whether or not the new method is capable of capturing the transition between the two regimes.

The fracture was discretized into 45 elements, with the length of the n–th element given by $L_n = (1.2)^{n-1}\,\mathrm{m}$; *i.e.,* the element lengths were 1.0 m, 1.2 m, 1.44 m, etc. Relatively small fracture gridblocks are needed near the inlet in order to accurately model the diffusive front, particularly at small times. The temperature was taken to be 20°C, and the initial capillary pressure was taken to be 1.013×10^5 Pa. This capillary pressure corresponds, through the capillary pressure functions, to an initial matrix saturation of 0.6765, and an initial fracture saturation of 0.0004.

We have solved this problem using TOUGH with the new dual-porosity module to perform the fracture/matrix interaction calculations, and also using TOUGH without the source/sink expressions, but with explicit discretization of the matrix rock adjacent to the fracture. When solving the problem with explicit discretization of the fracture and matrix regions, the matrix elements must be extended sufficiently far into the the formation so as to effectively simulate a semi-infinite region. This distance will depend on the total elapsed time of the simulation. In the example simulation, which covered an elapsed time of 10^8 s (about 3 years), the matrix elements were extended about 20 m away from the fracture. This was achieved using 20 matrix gridblocks in the direction transverse to the fracture, with the thickness of the n–th gridblock given by $L_n = (2.0)^n \times 10^{-5}$ m. The total length of matrix gridblocks in the direction normal to the fracture was therefore equal to 21.97 m. Note that very small matrix gridblocks are needed near the fracture in order to accurately resolve the saturation fronts in the matrix at small times.

The instantaneous flowrate of liquid into the fracture at the inlet is shown in Fig. 1. At early times, no appreciable leakage has taken place into the relatively impermeable matrix, and the flow field is essentially that of diffusive flow along the fracture. In this regime the flux into the fracture at the inlet decays as $t^{-1/2}$. As time progresses, the wetted interface area between the fracture and the matrix rock increases, and the effect of leakage becomes more important. The overall flowrate into the fracture gradually changes from a $t^{-1/2}$ variation to a $t^{-1/4}$ variation, as was predicted theoretically by Nitao and Buscheck (1991). The saturation profiles in the fracture are plotted in Fig. 2, at elapsed times of 10^4 s, 10^6 s, and 10^8 s. At each time, there is close agreement between the saturation profile predicted by the new lumped-parameter method, and that predicted using a discretized matrix.

The amounts of CPU time needed for the simulations, which were performed on a Solbourne (Series 5) computer, are shown in Table 1. In each case the simulation was carried out to 10^8 s, starting with an initial timestep of 0.01 s, and with no restrictions placed on the timestep growth. The lumped-parameter approach required about 70% fewer timesteps than did the fully-discretized solution, which is to say it allowed, on the average, timesteps that were about 3.38 times larger. The total number of Newton-Raphson iterations needed by the fully-discretized simulation was about 3.04 times greater than that needed by the lumped-parameter method. The savings in CPU time for the new method was about 96%, which corresponds to a 25-fold increase in speed. This reflects both an increased speed per iteration, and a need for a fewer total number of iterations to reach the desired total simulation time of 10^8 s.

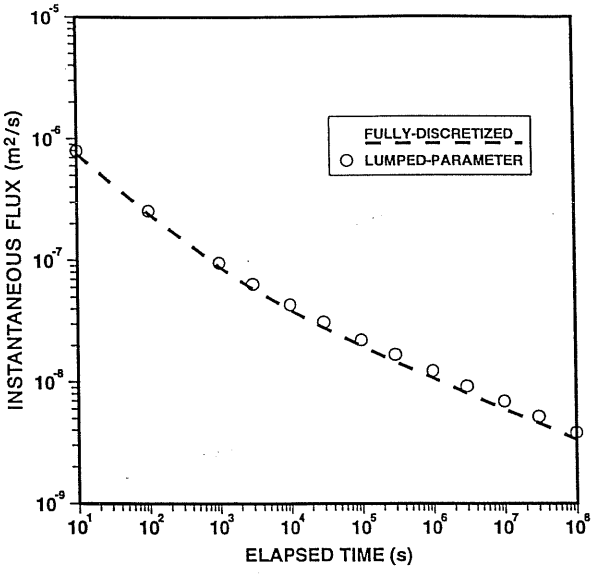

Fig. 1. Instantaneous flux into leaky fracture.

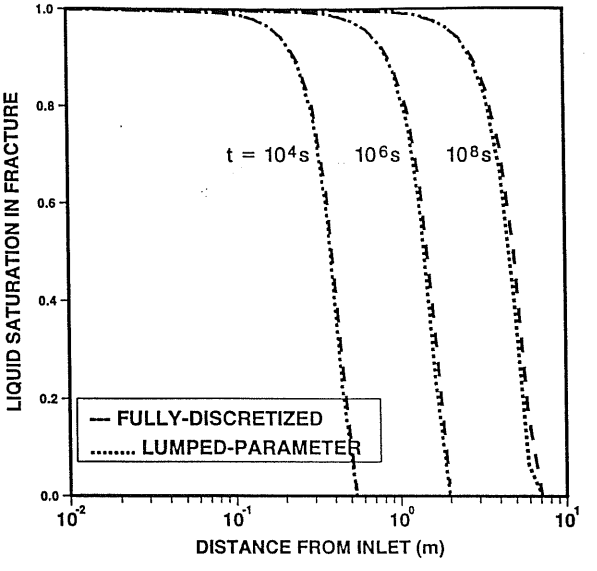

Fig. 2. Saturation profile in the fracture.

Table 1. CPU times for the problem of horizontal flow along a fracture located in a permeable formation. Both simulations were conducted with TOUGH running on a Solbourne (Series 5) computer.

	Fully-Discretized	Lumped-Parameter
# Fracture elements	45	45
# Matrix elements	45×20=900	0
Total # elements*	946	46
Timesteps	624	185
Iterations	3205	1053
CPU time (s)	20901	823

* Including one boundary element

Conclusions

A new lumped-parameter formulation has been developed for unsaturated flow in dual-porosity media. Fluid flow from the fracture network into the matrix blocks is modeled by a nonlinear equation, which can be thought of as a nonlinear extension of the Warren-Root equation. This expression for fracture/matrix flow has been incorporated into a module that is compatible with the TOUGH simulator, to act as a source/sink term for the fracture elements. The use of the modified code has been demonstrated on the problem of flow along a single horizontal fracture in a permeable rock, under constant-head boundary conditions. The new method gives very close agreement with simulations carried out by explicitly discretizing the matrix blocks, while yielding a substantial savings in CPU time. The new method also permits a simplification in the process of creating the mesh and the TOUGH input file. Other examples, and more details, can be found in Zimmerman et al. (1995).

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