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3-D NUMERICAL MODELING OF FRESHWATER LENS ON ATOLL ISLANDS

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ABSTRACT

Freshwater lenses are the major sources of water supply in many Pacific and Indian Ocean atoll islands. The thickness of lens is usually estimated based on the width and average hydraulic conductivity of the island and the rate of meteoric water recharge by using the Dupuit-Ghyben-Herzberg (DGH) methods. In this study, a series of TOUGH2 hydrology models are developed to determine the effects of various controls on the size of the freshwater lens and the thickness of the mixing zone. The results indicate that the structure of the mixing zone is strongly dependent on the physical dimension/shape of the island. A highly simplified representation of major bank margin fractures adequately reproduces the field observations, confirming the theory that fractures enhance freshwater/seawater mixing in atoll island groundwater systems.

1. INTRODUCTION

The difficulty in studying a regional aquifer system containing a narrow mixing zone between freshwater and seawater with both density driven fluid flow and solute dispersion models is twofold.

1. The applicability of the Fickian dispersion theory at the regional scale has been challenged and values of field dispersion parameters are neither well known nor may they be directly measured. The physical basis of a particular concentration gradient zone structure may be studied only after careful fitting of simulated hydraulics to the field conditions.

2. Excessively coarse computational grid, inefficiencies in moving boundary tracking, and inadequate numerical representation of density driven fluid flow have rendered many transport codes ineffective in simulating the movement of sharp concentration gradient fronts on a regional scale.

Traditionally, the immiscible interface condition has been adopted in transport simulation of freshwater/seawater systems containing portions of narrow mixing zones. This was the approach taken by Ghyben [1888] and Herzberg [1901] in their pioneering work on the dynamics of insular and coastal systems. They determined a relationship between the shape of the freshwater/seawater interface and its position, based on the density difference between these waters. This relationship is expressed by the classic Ghyben-Herzberg equation

$$z = (\rho f/ \rho s - \rho f)h, \qquad (1)$$

where z is the distance below sea level to an immiscible freshwater/seawater interface, h is the distance above sea level to phreatic surface, and ρs and ρf are the seawater and freshwater densities respectively.

The Ghyben-Herzberg approximation assumes that the fresh groundwater lens is in hydrostatic equilibrium with the seawater, i.e. no tidal stress effects. The thickness of a Ghyben-Herzberg lens is treated explicitly by the Dupuit-Ghyben-Herzberg (DGH) analysis (Bear [1972]; Fetter [1972]; Vacher [1988]; Budd & Vacher [1991]), in which the Ghyben-Herzberg Principle is combined with Darcy's Law, the continuity equation, and the Dupuit assumptions of horizontal flow. An elegant and useful correlation results from this analysis

$$H^2 = R(\alpha + 1)a^2/4K \qquad (2)$$

where H is the thickness of the lens (from water table to interface), a is the width across a infinite-strip island (2-D approximation), and α is the freshwater to saltwater density-difference ratio. The immiscibility condition obviously is not a realistic interfacial condition; the boundary between freshwater and seawater is not a sharp interface but rather a broad transition/mixing zone of brackish water (Figure 1). Recent field studies and variable-density flow modeling of large coastal type flow systems with large transition zones have placed the theoretical interface predicted by the DGH analysis on the upper half of the mixing zone (Todd & Meyer [1971], Vacher [1978], Kwiatkowski [1988]).

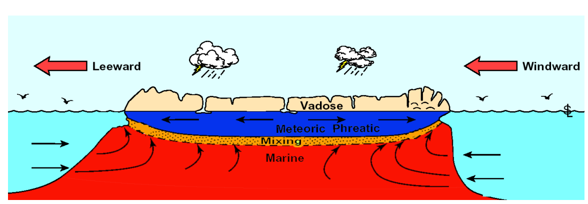

Figure 1. Diagenetic/hydrozone model for the isolated platform during sea level lowstand (modified from Moore (2001))

Although hydrogeologic numerical studies of atoll island groundwater systems have been conducted for many decades now, the constraints imposed by numerical instability, significantly large CPU time and memory requirement, and an inadequately detailed knowledge of the 3-D distribution of key hydraulic/physical parameters have restricted the vast majority of the research effort to two dimensional systems.

The dynamic groundwater system of an atoll island includes the flow and mixing of variable density fluids. There is no stream development on atolls, therefore the abundance of rainfall and island surface features basically control the amount of recharge to the freshwater lens. Long-term average recharge, island hydraulic conductivity, and bulk porosity control the average water levels and storage in the freshwater lens, whereas short-term tidal fluctuations mainly control the mixing processes. By excluding tidal stresses in the current model, the values of mixing zone thickness determined in this study would correspond to the lower limits of mixing zone thickness estimate.

The primary purpose of this work is to investigate relations among lens responses, aquifer parameters, and boundary conditions for a generalized, rather than site specific, shallow atoll island. Some of the questions to be addressed in this numerical study are how the geometric aspect ratio of a shallow atoll island affects the structure of the freshwater lens and how the formation of fracture networks on the platform margins impacts the freshwater/seawater mixing process.

2. MODEL DEVELOPMENT

Mesh Design

Since this study deals with generic atoll islands, a series of computational meshes were generated to represent atoll islands of various geometries (Figure 2). Meshes in Figure 2 (a)-(c) and (d)-(e) are shown with 6 and 22 times vertical exaggeration respectively. The ratio of the two principle axes, R1/R2, is reduced from 2.64 to 1.0 for meshes (a)-(c) with the shorter axis, R2, kept constant at 1,600 m. The island/ocean boundary is modeled with a 45 degree dip with respect to the horizon. Symmetry conditions were applied so only a quarter of the atoll island was computed in each case. Mesh in Figure 2(d)-(e) represents a more realistic atoll island geometry with approximate dimensions of 3,600 m by 1,800 m at its base (80 m below sea level). Vertical (z-axis) element spacings are the same to a depth of 8 m. Below this depth, vertical grid spacing increases successively downward (Figure 3). A more gradual, stepwise increase in depth with distance from one side of the shore was designed to capture the topological differences between the windward- and leeward-sides of an atoll island as depicted in Figure 1.

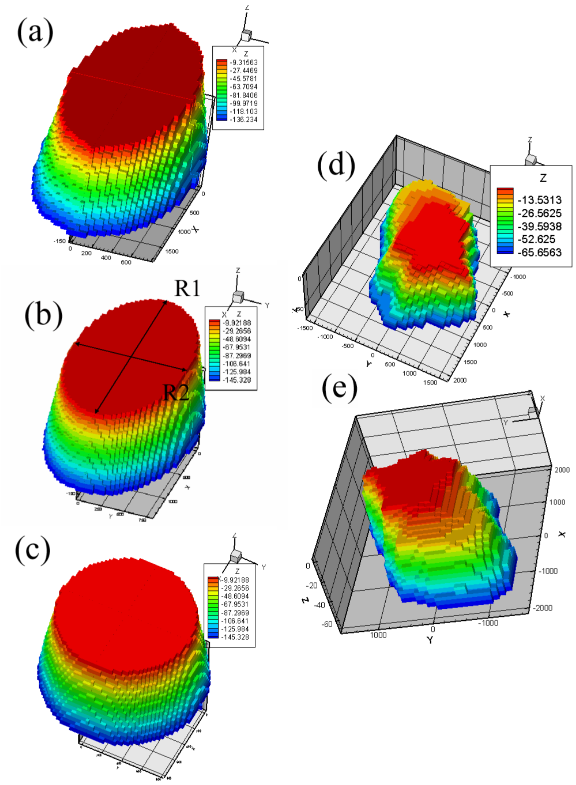

Figure 2. Computational meshes used. (a) R1/R2 = 2.64; (b) R1/R2 = 1.45; (c) R1/R2 = 1.0; (d) & (e) different views of an irregular island geometry.

Boundary Conditions

Specified pressure values were assigned to all of the grids that border the ocean, and were made equivalent to hydrostatic seawater with respect to sea level. A value for solute mass concentration equivalent to seawater was also assigned to each specified pressure grid. Freshwater sources (arrows in Figure 3) were assigned to grids representing the water table beneath the island's interior, where recharge enters the groundwater system. Heat sources were assigned to the bottom of the grid domain to simulate geothermal flux (50 mW/m^2). EOS7 was used for the simulations.

Input Data

Input data consisted of two parts: (1) distribution of aquifer characteristics across the mesh and (2) distribution of initial pressure and concentration values to the nodes of the mesh. The distribution of aquifer parameters across the mesh was as follows: the porosity value decreased gradually with increasing distance from island/ocean boundary to reflect the changes in depositional environment. Porosity variation as a function of depth was ignored (estimates using Athy's (1930) law had showed negligible porosity reduction with depth for shallow platforms); permeability values were assigned separately to island interior and to bank margins to study the effects of fracturing.

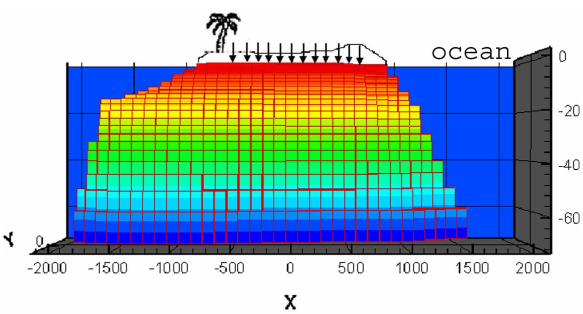

Figure 3. Mesh construction for area beneath atoll island and imposed boundary conditions.

3. MODELING RESULTS

More than a hundred numerical simulations were conducted to calibrate the modeling parameters and to benchmark the results against values reported in the literature. In this paper the focus is on simulations that investigate: (1) lens response to atoll island physical shape and size (2) lens response to bank margin fracturing. Simulations were run to a 'steady state' condition when changes in salinity and pressure profiles were negligible. The parameter values resulting from model calibration assigned to represent the generic atoll islands are listed in Table 1.

Lens Responses

In this study we define the lens beneath an atoll island as being a body of water with salt concentration less than that of seawater, and having a brackish mixing zone with dissolved solid composition ranging from 2.5% to 95% seawater, and a potable zone with solute concentrations below 2.5% seawater.

The 50% seawater isopleth along the short axis, R2, rather than the 2.5% seawater isopleth, was used in this study as an indicator of potable lens behavior because it represents the approximate center of mass of the transition zone and therefore is not strongly influenced by factors that cause dispersion. 2-D simulations conducted by Underwood et al. [1992] showed that the position of the 50% seawater isopleth was primarily controlled by the hydraulic conductivity of the aquifer and the recharge rate, and was largely independent of tidal fluctuations.

Table 1. Parameter Values Used to Describe Atoll Island Groundwater Systems

Grid	*Parameter*	*Value*	*Units*
(a)-(c) in Figure 2.	*Porosity*	*0.25*	*...*
	Horizontal Perm. Kx = Ky	*1.0*	*Darcy*
	Vertical Perm. Kz	*0.2*	*Darcy*
	Island Width	*1,600*	*m*
	Freshwater Recharge	*0.5*	*m/yr*
	Diff. Coeff.	*1.0e-9*	*m^2/s*
(d)-(e) in Figure 2	*Porosity (Platform interior to shore)*	*0.25 - 0.33*	*...*
	Horizontal Perm. Kx = Ky	*1.0*	*Darcy*
	Vertical Perm. Kz	*0.2*	*Darcy*
	Horizontal Fractured Zone Perm.	*1,000.0 (Parallel to bank margin)*	*Darcy*
	Vertical Fractured Zone Perm.	*1,000.0*	*Darcy*
	Freshwater Recharge	*0.5*	*m/yr*
	Diff. Coeff.	*1.0e-9*	*m^2/s*

Figure 4 shows the 50% seawater isopleth for atoll islands with different geometric aspect ratios. For the cases considered in this study, the depth of freshwater infiltration is observed to increase linearly with island length. This is an important result because it demonstrates that the linear dependence of lens thickness with island width predicted by the DGH analysis is not restricted to islands with very high aspect ratios.

The 2.5% seawater isopleth defines the reserve of potable groundwater beneath an atoll island, so it is extremely important in freshwater development and management considerations. Again, a monotonic increase in the depth of the seawater isopleth with island geometric aspect ratio is observed (Figure 5). An important distinction from the 50% seawater isopleth is that now the functional dependence is quadratic. Unlike the 50% seawater isopleth, however, the 2.5% seawater isopleth is expected to be strongly dependent on dispersive processes

(Underwood et al. [1992]), and hence should be highly sensitive to the vertical longitudinal dispersivity and the magnitude of the tidal fluctuations which are not considered in the current model. Therefore, the depth of the 2.5% isopleth in an actual atoll island of similar physical dimensions is expected to be lower than the values shown in Figure 5.

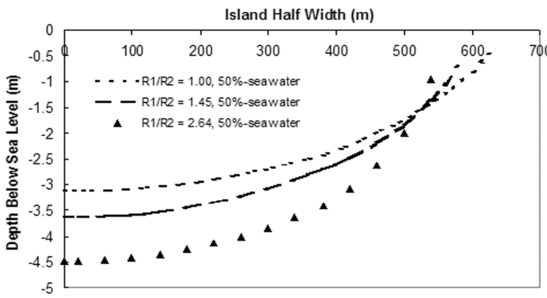

Figure 4. Position of the 50% seawater isopleth along the short axis, R2, for atoll islands with different geometric aspect ratios.

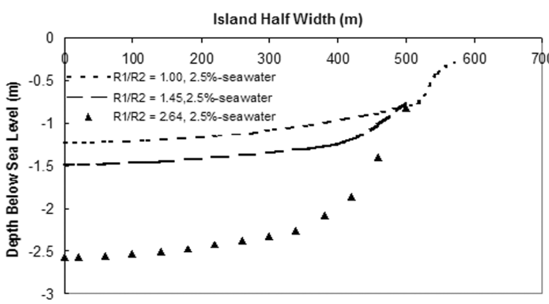

Figure 5. Position of the 2.5% seawater isopleth along the short axis, R2, for atoll islands with different geometric aspect ratios.

One interesting result from the simulations is the significant reduction in mixing zone thickness with increasing island geometric aspect ratio. In Figure 6 the depth of the 95% seawater isopleth can be observed to slowly level off as the R1/R2 value exceeds 1.45 except in the proximity of the island/ocean boundary. A higher R1/R2 ratio appears to alter the aquifer flow field in such a way that most of the flow exchange/mixing processes become confined to the thin coastal region of the platform (Figure 7).

The computed flow streamlines (Figure 8) indicates that the broadening of the mixing zone is confined at the potable limits by the meteoric water recharge and at the marine zone limits by the buoyant effects of the less dense mixing zone water, which cause upward and outward flow toward the discharge areas.

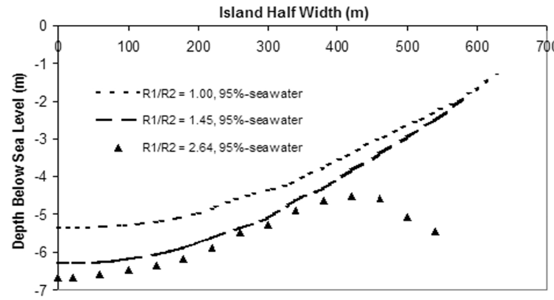

Figure 6. Position of the 95% seawater isopleth along the short axis, R2, for atoll islands with different geometric aspect ratios.

Effect of bank margin fracturing on freshwater lens

Large scale fracture systems are often found along the atoll island bank margins (Whitaker & Smart [1997]). In the absence of significant tectonic motion, the fracture planes generated by over-steepening of the platform flanks and lateral unloading are nearly vertical and without significant lateral or vertical displacement. In addition to providing a discharge route for meteoric, mixed, and geochemically altered saline groundwaters, these high permeability pathways are known to generate strong local recirculation along the fluid flow path. This gives rise to enhanced vertical mixing within the voids of the fracture system, as evidenced by the increasing mixing zone thickness observed in field studies.

The effect of these fracture networks is modeled by increasing the vertical permeability and the horizontal permeability in the direction parallel to the bank margins of the grid blocks in the vicinity of the island/ocean boundary. The simulation results are shown in Figure 9. Fracturing is shown to effectively reduce the size of the fresh water lens by increasing the thickness of the mixing zone, in agreement with field observations. Tidal pumping is expected to significantly enhance the mixing effects because the connected fracture system would provide a favorable pathway for tidal pulse propagation.

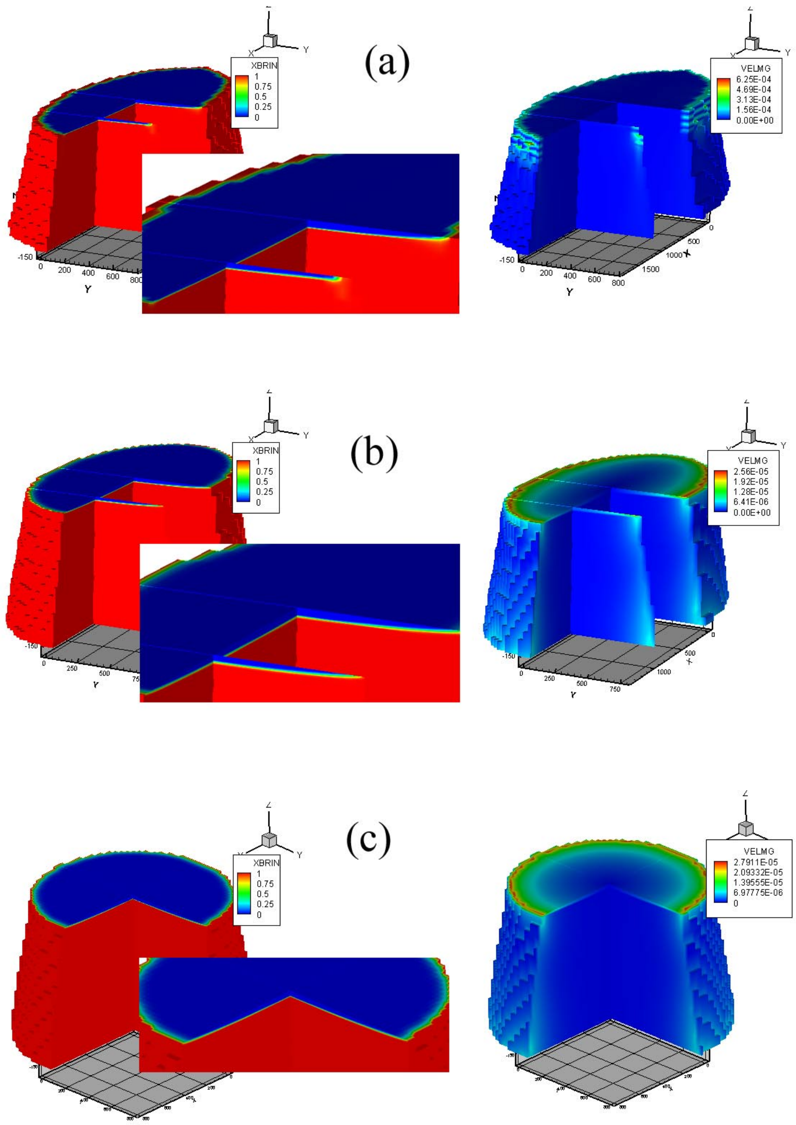

Figure 7. Effect of atoll island geometry on freshwater distribution and fluid flow speed. (a) R1/R2 = 2.64; (b) R1/R2 = 1.45; (c) R1/R2 = 1.0.

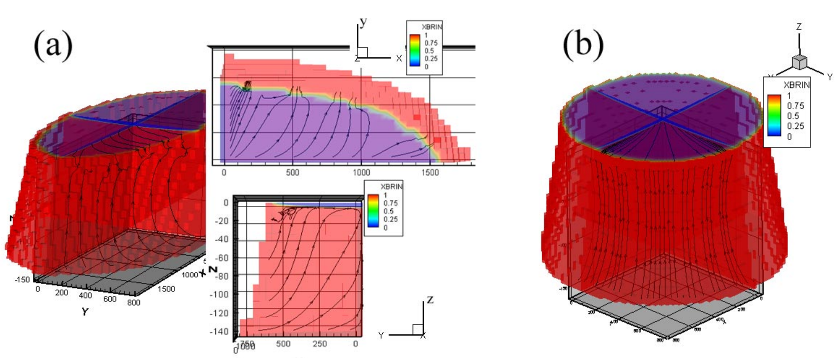

Figure 8. Flow Streamlines. (a) R1/R2 = 2.64; (b) R1/R2 = 1.0.

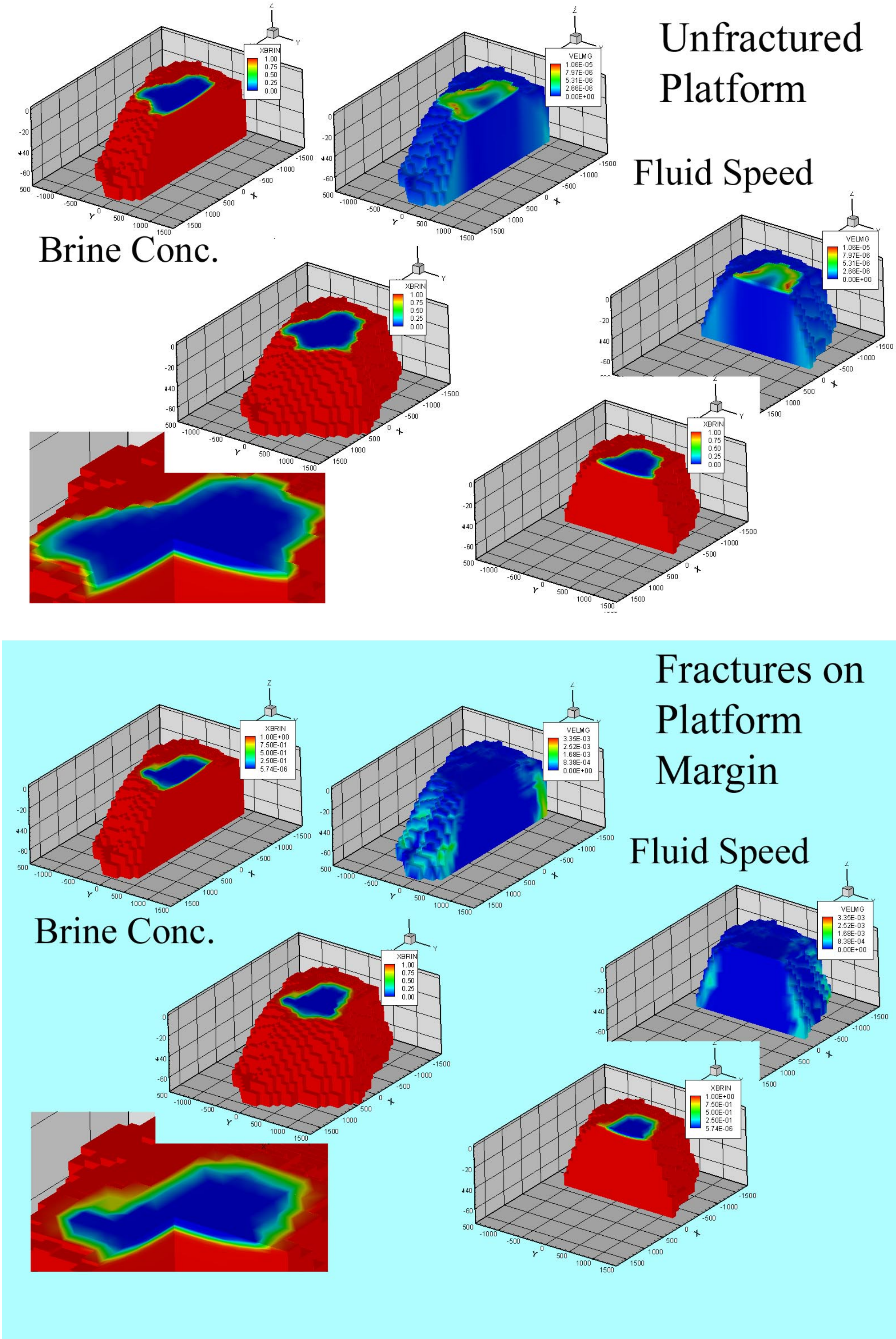

Figure 9. Effect of bank margin fracturing on freshwater lens.

4. CONCLUSION

The atoll island groundwater system has two components of flow: short-term vertical fluctuations that take place over small length scales (1m or less) and are driven by tidal stresses, and long-term average flow that is driven by meteoric water recharge and takes place over the scale of island width. Recharge is the source of fresh water that drives long-term groundwater flow, hydraulic conductivies control the flow of fresh water and the extent of freshwater-seawater mixing, and the bulk porosity determines the amount of water stored in the aquifer.

The simulation results indicate that the thickness of the mixing zone is reduced by an increase in the island geometric aspect ratio. By excluding tidal stresses in the calculations, the mixing zone thickness values reported in this study are expected to be lower than those found in actual atoll islands groundwater systems.

The bank margin fractures are found to increase the thickness of the mixing zone, in agreement with field observations. Tidal pumping, which was not included in this study, is expected to further enhance the mixing process which takes place within the voids of the fracture networks.

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